Some problems from HW9

Answers

Questions

Question 1. A solid object lying in the region in space between the cylinders $x = y^2$ and $x = 4 - y^2$ and the planes $z = 0$ and $z = x$ has density $\rho(x, y, z) = xy^2z$. Set up, but do not evaluate, an integral which computes the mass of this object.

Originally it was $\rho(x, y, z) = xyz$ but in class I realized that this density function is nonsense, because it would be negative at places inside the region. So I changed it to $\rho(x, y, z) = xy^2z$. It doesn't really change the work involved in the problem at all, but it made me feel better.

Below are brief answers to the worksheet exercises. If you would like a more detailed solution, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to questions

Question 1. The mass is just computed by integrating $\rho(x, y, z)\, dV$ over the region, so the task is just to figure out bounds of integration. The $dz\, dx\, dy$ order is the most convenient for this, and we get

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{y^2}^{4-y^2} \int_0^x x y^2 z \, dz \, dx \, dy.$$

I drew a picture of this region i nclass.